

## STATISTIC ANALYSIS OF ANNUAL TOTAL OZONE EXTREMES FOR THE PERIOD 1964-1988

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### ABSTRACT

Annual extremes of total column amount of ozone (in the period 1964-1988) from a network of 29 Dobson stations have been examined using the extreme value analysis.

The extremes have been calculated as the highest deviation of daily mean total ozone from its long-term monthly mean, normalized by the monthly standard deviations. The extremes have been selected from the direct-Sun total ozone observations only. The extremes resulting from abrupt changes in ozone (day to day changes greater than 20%) have not been considered.

The ordered extremes (maxima- in ascending way, minima - in descending way) have been fitted to one of three forms of the Fisher-Tippet extreme value distribution by the nonlinear least square method (Levenberg-Marguard method).

We have found that the ordered extremes from a majority of Dobson stations lie close to Fisher-Tippet type III.

The extreme value analysis of the composite annual extremes (combined from averages of the annual extremes selected at individual stations) has shown that the composite maxima are fitted by the Fisher-Tippet type III and the composite minima by the Fisher-Tippet type I. The difference between the Fisher-Tippet types of the composite extremes seems to be related to the ozone downward trend.

Extreme value prognoses for the period 1964-2014 (derived from the data taken at: all analyzed stations, the North American, and the European stations) have revealed that the prognostic extremes are close to the largest annual extremes in the period 1964-1988 and there are only small regional differences in the prognoses.

### 1. INTRODUCTION

Analyses of both ground-based [IOTP, 1990] and satellite ozone data [Stolarski et al., 1991] indicate statistically significant total ozone decline.

The potential effects of an increase in solar ultraviolet radiation on the biosphere have stimulated much of the concern about ozone depletion. Then, the estimation of future ozone changes is of special interest for human life.

In this paper we examine possibilities to implement the theory of extreme value to infer future total ozone extremes. Many extreme value analyses of

hydrological and meteorological variables have been undertaken in the past (for review, see Tabony, 1983). The method provides estimation of the highest and the lowest values which meteorological (or other) variable is likely to attain in a given number of years. The extreme value prognoses are based on the assumption that all factors determining changes in a prognostic variable have already appeared during the period of observations.

### 2. DATA

Total ozone extremes in the period 1964-1988 have been selected from a network of 29 Dobson stations. The names of the stations, index in the World Ozone Data Center, locations and the data periods are listed as follows:

1	Lerwick,	U.K.	43,	60°N,	1°W;	1/64-11/88
2	St.Petersburg	Russia	42,	60°N	30°E;	8/68-12/88
3	Churchill,	Canada	77,	59°N,	94°W;	1/65-12/88
4	Edmonton,	Canada	21,	54°N,	114°W;	1/64-12/88
5	Belsk,	Poland,	68,	52°N,	21°E;	1/64-12/88
6	Bracknell,	U.K.	102,	51°N,	1°W;	1/69-12/88
7	Uccle,	Belgium	53,	51°N,	4°E;	2/71-12/88
8	Hradec K.,	Czech.	96,	50°N,	16°E;	1/64-12/88
9	Hohenpeiss.,	Germany	99	48°N,	11°E;	1/67-12/88
10	Caribou,	U.S.A.	20,	47°N,	68°W;	1/64-12/88
11	Arosa,	Switzerland	35,	47°N,	10°E;	1/64-12/88
12	Bismarck,	U.S.A.	19,	47°N,	101°W;	1/64-12/88
13	Toronto,	Canada	65,	44°N,	79°W;	1/64-12/88
14	Sapporo,	Japan	12,	43°N,	141°E;	1/64-12/88
15	Rome,	Italy	55,	42°N,	12°E;	1/64-12/88
16	Boulder,	U.S.A.	67	40°N,	105°W;	1/64-12/88
17	Cagliari,	Italy	38,	39°N,	9°E;	1/64-12/88
18	Wallops Is.,	U.S.A.	107,	38°N,	76°W;	1/70-12/88
19	Nashville,	U.S.A.	106,	36°N,	87°W;	1/64-12/88
20	Tateno,	Japan	14,	36°N,	140°E;	1/64-12/88
21	Srinagar,	India	13,	34°N,	75°E;	2/64-12/88
22	Kagoshima,	Japan	7,	32°N,	131°E;	1/64-12/88
23	Quetta,	Pakistan	11,	30°N,	67°E;	8/69-12/88
24	Cairo,	Egypt	152,	30°N,	31°E;	11/74-12/88
25	Naha,	Japan	190,	26°N,	128°E;	4/74-12/88
26	Mauna Loa,	U.S.A.	31,	19°N,	156°E;	1/64-12/88
27	Huancayo,	Peru	110,	12°S,	75°W;	2/64-12/88

28 Samoa, U.S.A. 191, 14°S, 171°W; 1/76-12/88  
 29 Aspendale, Australia 26, 38°S, 145°E; 1/64-12/82

For each station, the annual extremes in total column amount of ozone have been calculated from the daily averages of total ozone published in Ozone Data for the World (ODW) journals. However, International Trend Panel-1988 [IOTP, 1990] have found strong incongruities in the published data of many ground-based stations due to instrument calibrations.

A provisionally revised data set (1957-1986) was prepared by the Panel, with the corrections being applied to the monthly averages of total ozone values.

Making use of the Panel's re-evaluated total ozone monthly means we convert all daily averages of total ozone values (calculated before Jan. 1987) to the provisionally corrected ones in the following way:

$$O_{3new}(i, j, k, l) = O_{3old}(i, j, k, l) \cdot \frac{M1(j, k, l)}{M2(j, k, l)}$$

where:  $O_{3new}(i, j, k, l)$ ,  $O_{3old}(i, j, k, l)$  - daily mean of total ozone on  $i$ -th day,  $j$ -th month,  $k$ -th year and at  $l$ -th Dobson station, provisionally revised and unrevised, respectively,  $M1(j, k, l)$  - revised monthly mean of total ozone at  $l$  station in  $j$ -th month and  $k$ -th year (data from IOTP, 1990),  $M2(j, k, l)$  - monthly mean of total ozone derived from ODW data.

Provisionally corrected daily averages of total ozone values are transformed to the departures from the long-term monthly means (1964-1988) expressed in percents of the long-term monthly standard deviation. Finally, the annual extremes have been selected from the dimensionless daily averages of total ozone values.

ODW data from only a few stations (Belsk, Sapporo, Kagoshima, Tsukuba, Naha) could be accepted without any adjustment of the data, because original reported results of total ozone measurements were re-evaluated (and published in ODW) on daily basis taking into account all instrument's calibrations. Our correction method provides only a crude estimation of daily mean total ozone.

ODW data contain results from the various types of ozone observations among these the direct-Sun ozone observations provide the highest accuracy. Taking this into consideration, the extremes have been selected only from the direct-Sun observations. The extremes, which were formed as a result of abrupt changes in the ozone pattern (day-to-day change in total ozone more than 20%), have not been considered. Therefore, the extremes in total ozone, examined in this paper, are caused by processes with a time scale larger than a few days. Moreover, in the case when the day-to-day change in total ozone is very large there is a larger likelihood that this change is a result of erroneous observations.

### 3. THE EXTREME VALUE ANALYSIS

The theory of extreme value was first developed by Fisher and Tippet, [1928]. They showed that extremes taken from a sample containing  $N$  observations converge asymptotically towards one of only three forms as  $N$  increases. These are called Fisher-Tippet Type I, Type II and Type III. A general solution of the extreme value problem was obtained by Jenkinson [1955] in the form:

$$X = X_0 + a \frac{1 - \exp(-k Y)}{k} \quad [1]$$

where:  $X$  - extremes (ordered in ascending way),  
 $Y$  - reduced variate calculated from the equation:

$$F(X) = \exp(-\exp(-Y)),$$

$F(X)$  - empirical cumulative probability function,  $X_0$ ,  $a$ , and  $k$  constants to be calculated.

Fisher-Tippet types I, II, and III are characterized by their different shapes when plotted on a graph  $X$  against  $Y$  (see Fig.1). Type I corresponds to  $k = 0$  and forms a straight line. Type II (bounded below but not above) and Type III (bounded above but not below) correspond to  $k < 0$  and  $k > 0$ , respectively.

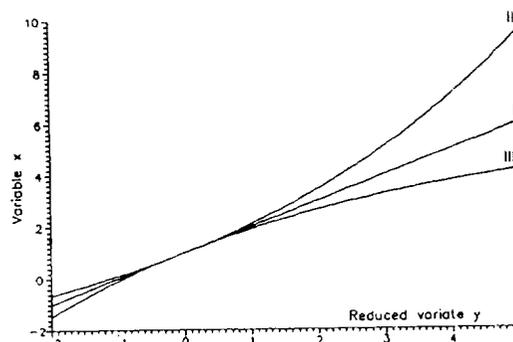


Fig. 1. Fisher-Tippet distribution types I, II, III

### 4. RESULTS

In order to use extreme value method, the empirical cumulative distribution function of annual extremes (ECDFAE) has to be constructed. The annual extremes are ordered from the lowest  $X_1$  (for minima we use absolute values) to the highest  $X_m$ , where  $m$  is the number of annual extremes. ECDFAE could be calculated by various formulas, we have chosen Beard's formula [Beard, 1943]:

$$F(X_i) = (i - 0.31) / (m + 0.38)$$

Finally, one of the three Fisher-Tippet types has to be fitted to a set of  $(X_i, Y_i)$  points,

$$Y_i = -\ln(-\ln(F(X_i)))$$

In practice, after plotting the set of  $(X_i, Y_i)$  points on a graph  $X$  against  $Y$ , the Fisher-Tippet type of ECDFAE is determined "by eye" method and the parameters describing precisely the pattern of ECDFAE are calculated assuming that the type of ECDFAE is known. However, in our paper we use the nonlinear least square method (Levenberg-Margardt method) to find the model's constants:

$X_0$ ,  $a$ , and  $k$ .

Then, both the values of  $k$  and the standard error of  $k$  let us determine the type of ECDFAE.

In Fig.2, the estimates of  $k$  from individual stations extremes data are presented. It is seen that Fisher-Tippet type III dominates.

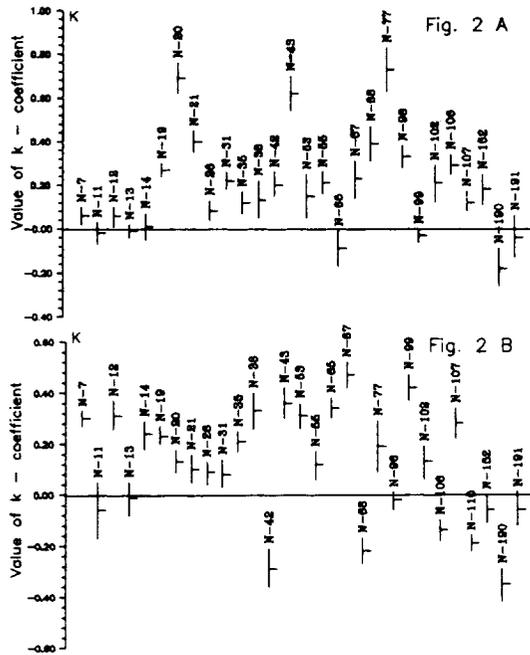


Fig.2. Values of  $k$  for annual maxima (Fig.2 A) and for annual minima (Fig. 2 B) derived from the annual extremes taken at the selected Dobson stations. Number above each point corresponds to the ODW station index. Vertical bars represent standard errors

When, the extremes lie close to the physically imposed bound, a type III distribution of the extremes maybe expected, but when the observed extremes fall far from their limit, they may appear to be fitted by types I or II. However, type III and II may appear erroneously if the extremes are selected from small population (too small number of direct-Sun measurements per year) or the data contain samples from various populations (extremes are caused by more than one mechanism). Type II can be form misleadingly if the data are contaminated by only a few very large extremes coming from erroneous observations.

Then, to determine the type of extremes from individual station data correctly we need to analyze a cause of the extremes appearance.

Examination of the composite extremes, combined from the annual extremes taken at each station, is an attempt to eliminate the observation errors. In Fig.3 we present result of extreme value analysis applied to the composite annual total ozone extremes.

Maxima are well fitted by Fisher-Tippet type III distribution,  $k = 0.46 \pm 0.06$ . The minima (absolute values) lie close to Fisher-Tippet type I distribution,  $k = -0.02 \pm 0.06$ . These findings imply that the future maxima with the values much larger than the largest maximum registered up to 1988 will not be expected (the highest maximum in the period 1964-1988 lies close to upper limit imposed by Fisher-Tippet III type), while more negative minima will be recorded (total ozone minima seem to be unbounded).

The difference between the Fisher-Tippet types obtained for the annual maxima and minima seems to

be related to decreasing tendency in atmospheric ozone. The ozone extremes have been calculated as the highest annual departures from the long-term (1964-1988) means. In the future, larger maxima (deeper minima) than the ones recorded up to 1988 will be unlikely (likely) because the negative trends in ozone attenuate the maxima (amplify the minima) relative to the ozone mean level in the period 1964-1988.

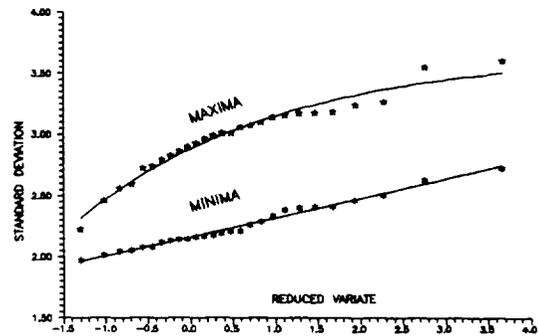


Fig.3. Empirical cumulative probability function of annual extremes (for minima we use absolute values) combined from the annual total ozone extremes taken at 29 Dobson stations.

The extreme value method provides the possibility to estimate the highest and the lowest values, which any meteorological (or other) variable is likely to attain in a given number of years. The estimation is obtained from the extrapolation of Fisher-Tippet distribution at a reduced variate value, which corresponds with a chosen period of prognosis.

Tab.1 shows results of extreme value prognosis in the period 1964-2014 for all Dobson stations listed in section 2 and for 10 European and 8 North American stations. We examine both provisionally revised data and non corrected ODW data.

Table 1. Extreme value prognoses (expressed in the standard deviations units) in the period 1964-2014 for 29 Dobson stations and for the European and the North American stations. Standard errors of the estimates in parentheses.

Class	D A T A			
	REVISED		NON-REVISED	
	MIN	MAX	MIN	MAX
All stat.	-2.79(0.02)	3.53(0.04)	-2.68(0.03)	3.55(0.04)
Europe	-2.80(0.03)	3.91(0.05)	-2.81(0.04)	3.90(0.05)
N.Amer.	-2.82(0.04)	3.47(0.04)	-2.79(0.03)	3.53(0.05)

Comparing the extreme value prognoses for the analyzed groups of ozone stations we have found that; provisionally revised data and non corrected ODW data give similar result, there are small regional differences in the prognoses. Prognoses for minima (absolute values) are lower than for maxima (this reflects positive asymmetry of the parent distribution).

In the period 1964-1988, the largest maximum (about 3.5 standard deviation) and minimum (about -2.6 standard deviation), calculated for the group of all the analyzed stations (see Fig.3), differ only slightly from the prognostic extremes for the period 1964-2014.

We estimate that 1 standard deviation for the group of all the analyzed stations equals about 10 Dobsons. Then, the predicted largest minimum (for the period 1964-2014) is only a few Dobsons lower than that observed in the period 1964-1988. Therefore, a downward trend in total ozone (about 1.5 % per decade for the region 30°N-60°N in the period 1970-1988, Krzyścin, 1992) will influence only a little the future largest extremes.

Almost all studies of the long-term variations in atmosphere ozone (especially the trend models) were limited to analyses based on the monthly means or the annual means of total ozone. Our paper reveals that the extreme value analysis provides a useful tool for the detection long-term changes in atmospheric ozone. The reliability of climatological statements based on the analysis of ozone extremes is clearly dependent upon quality of the data. Then, elimination of spurious extremes from the data seems to be the most important part of the analysis.

#### ACKNOWLEDGMENTS

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